DIFFERENTIATION AND INTEGRATION

(1) Find the derivative of the following functions:

(a) $f(x) = x^{33} + 4x^{12} + 3x^4 + \sqrt{42}x$

(b) $g(y) = \sin y + \cos y + \tan y$

(c) $h(t) = (t^4 + 6)\sin t$

(d) $\phi(x) = \dfrac{1}{\sqrt[3]{x + \sqrt[3]{x}}}$

(e) $\psi(t) = \sin^2\left(\dfrac{t^3+1}{t^2+2t}\right)$

(f) $g(x) = \ln\ln\ln x$

(g) $h(x) = \cos(e^{\sin x})$

(h) $F(t) = e^{t\cos^2 t}$

(i) $y = \sqrt{1 + e^x \ln(x^2+1)}$

(j) $p(x) = \left[\ln(x^3 + 2x + 2)\right]^4$

(2) Calculate the first and second derivative of the following functions

(a) $y = \dfrac{x^2 - 2\sqrt{x}}{x}$

(b) $v = \sqrt[5]{u^3} - 4\sqrt[7]{u^{11}}$

(3) Find the equation of the tangent line and the normal line to the curve

$$y = \sqrt{1 + x^3} \text{ at } x = 2.$$

(4) Evaluate the following indefinite integrals:

a) $\displaystyle\int x\sqrt{x}\, dx$

b) $\displaystyle\int (\cos x - 2\sin x)\, dx$

c) $\displaystyle\int (1-t)(2+t^2)\, dt$

d) $\displaystyle\int x\sin(x^2)\, dx$

e) $\displaystyle\int \frac{x}{(x^2+1)^3}\, dx$

f) $\displaystyle\int \sin^4 x\cos x\, dx$

g) $\displaystyle\int \frac{\cos\left(\frac{1}{x}\right)}{x^2}\, dx$

h) $\displaystyle\int x(2x+3)^6\, dx$

i) $\displaystyle\int \sin x\cos(\cos x)\, dx$

j) $\displaystyle\int \frac{e^{1/x^2}}{x^3}\, dx$

k) $\displaystyle\int e^x \sqrt[5]{9 + e^x}\, dx$

l) $\displaystyle\int \frac{e^{3x+2}}{\sqrt{1-e^{6x}}}\, dx$

m) $\displaystyle\int x^2 e^{-3x}\, dx$

n) $\displaystyle\int e^\theta \sin\theta\, d\theta$

(5) Evaluate the following indefinite integrals

a) $\displaystyle\int_0^1 \sqrt[3]{1+7x}\, dx$

b) $\displaystyle\int_0^1 (8x^3 + 3x^2)\, dx$

c) $\displaystyle\int_0^2 y^2\sqrt{1+y^3}\, dy$

d) $\displaystyle\int_0^3 \frac{x^4+1}{x^5+5x+1}\, dx$

e) $\displaystyle\int_e^9 \frac{dx}{x\ln x}\, dx$

Tables of Integrals

- $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + C \; ; \; n \neq -1$
- $\int \dfrac{dx}{x} = \ln |x| + C$
- $\int e^x \, dx = e^x + C$
- $\int \sin x \, dx = -\cos x + C$
- $\int \cos x \, dx = \sin x + C$
- $\int \tan x \, dx = \ln |\sec x| + C$
- $\int \sec^2 x \, dx = \tan x + C$
- $\int \csc^2 x \, dx = -\cot x + C$
- $\int \sec x \tan x \, dx = \sec x + C$
- $\int \csc x \cot x \, dx = -\csc x + C$
- $\int u \, dv = uv - \int v \, du$
- $\int_a^b u \, dv = uv \Big|_a^b - \int_a^b v \, du$